26, 22, 46, 60	Maggie
25, 20, 45, 59	Sophie
23, 18, 44, 58	Alex
21, 16, 43, 57	Sylvie
19, 14, 42, 56	Jake
17, 34, 41, 55	Clare
15, 33, 40, 54	Eliot
13, 32, 39, 53	Ben
12, 31, 38, 52	Isaiah
11, 29, 37, 51	Fabrizzio
10, 27, 36, 50	Shaikwa

Pretty Mild

10. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

11. $\csc\theta - \sin\theta = \cot\theta\cos\theta$

12. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

13. $\dfrac{\tan\theta\cot\theta}{\csc\theta} = \sin\theta$

15. $\sin^2\theta(1 + \cot^2\theta) = 1$

17. $\sin t\tan t = \dfrac{1 - \cos^2 t}{\cos t}$

19. $\dfrac{\csc^2 t}{\cot t} = \csc t\sec t$

21. $\dfrac{\tan^2 t}{\sec t} = \sec t - \cos t$

23. $\dfrac{1 - \cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

25. $\dfrac{\sin t}{\csc t} + \dfrac{\cos t}{\sec t} = 1$

26. $\dfrac{\sin t}{\tan t} + \dfrac{\cos t}{\cot t} = \sin t + \cos t$

27. $\tan t + \dfrac{\cos t}{1 + \sin t} = \sec t$

29. $1 - \dfrac{\sin^2 x}{1 + \cos x} = \cos x$

31. $\dfrac{\cos x}{1 - \sin x} + \dfrac{1 - \sin x}{\cos x} = 2\sec x$

32. $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x} = 0$

33. $\sec^2 x\csc^2 x = \sec^2 x + \csc^2 x$

34. $\csc^2 x\sec x = \sec x + \csc x\cot x$

14. $\dfrac{\cos\theta\sec\theta}{\cot\theta} = \tan\theta$

16. $\cos^2\theta(1 + \tan^2\theta) = 1$

18. $\cos t\cot t = \dfrac{1 - \sin^2 t}{\sin t}$

20. $\dfrac{\sec^2 t}{\tan t} = \sec t\csc t$

22. $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$

Medium to Hot

36. $\dfrac{\csc x - \sec x}{\csc x + \sec x} = \dfrac{\cot x - 1}{\cot x + 1}$

37. $\dfrac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$

38. $\dfrac{\tan^2 x - \cot^2 x}{\tan x + \cot x} = \tan x - \cot x$

39. $\tan^2 2x + \sin^2 2x + \cos^2 2x = \sec^2 2x$

40. $\cot^2 2x + \cos^2 2x + \sin^2 2x = \csc^2 2x$

41. $\dfrac{\tan 2\theta + \cot 2\theta}{\csc 2\theta} = \sec 2\theta$

42. $\dfrac{\tan 2\theta + \cot 2\theta}{\sec 2\theta}$

43. $\dfrac{\tan x + \tan y}{1 - \tan x\tan y} = \dfrac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y}$

44. $\dfrac{\cot x + \cot y}{1 - \cot x\cot y} = \dfrac{\cos x\sin y + \sin x\cos y}{\sin x\sin y - \cos x\cos y}$

45. $(\sec x - \tan x)^2 = \dfrac{1 - \sin x}{1 + \sin x}$

46. $(\csc x - \cot x)^2 = \dfrac{1 - \cos x}{1 + \cos x}$

47. $\dfrac{\sec t + 1}{\tan t} = \dfrac{\tan t}{\sec t - 1}$

49. $\dfrac{1 + \cos t}{1 - \cos t} = (\csc t + \cot t)^2$

Probably Spicy

50. $\dfrac{\cos^2 t + 4\cos t + 4}{\cos t + 2} = \dfrac{2\sec t + 1}{\sec t}$

51. $\cos^4 t - \sin^4 t = 1 - 2\sin^2 t$

52. $\sin^4 t - \cos^4 t = 1 - 2\cos^2 t$

53. $\dfrac{\sin\theta - \cos\theta}{\sin\theta} + \dfrac{\cos\theta - \sin\theta}{\cos\theta} = 2 - \sec\theta\csc\theta$

54. $\dfrac{\sin\theta}{1 - \cot\theta} - \dfrac{\cos\theta}{\tan\theta - 1} = \sin\theta + \cos\theta$

55. $(\tan^2\theta + 1)(\cos^2\theta + 1) = \tan^2\theta + 2$

56. $(\cot^2\theta + 1)(\sin^2\theta + 1) = \cot^2\theta + 2$

57. $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = 2$

58. $(3\cos\theta - 4\sin\theta)^2 + (4\cos\theta + 3\sin\theta)^2 = 25$

59. $\dfrac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

60. $\dfrac{\sin x + \cos x}{\sin x} - \dfrac{\cos x - \sin x}{\cos x} = \sec x\csc x$

You will create a Key for ~~four~~ problems it must be checked by a partner. You may work together and please ask questions!

Jake Wilson

#14:

$$= \frac{\csc^2 x}{\cot x}$$

$$= \frac{1 + \cot^2 x}{\cot x}$$

$$= \frac{1}{\cot x} + \frac{\cot^2 x}{\cot x}$$

$$= \tan x + \cot$$

$$= \frac{\cos x \cot x}{\sin x \cot x} \cdot \frac{\sin x}{\sin x} + \frac{\sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$= \boxed{\sec x \cdot \csc x}$$

#14:

$$\frac{\cos x \cdot \sec x}{\cot x} = \tan x$$

$$= \frac{\frac{1}{\cos} \cdot \frac{\cos}{1}}{\cot x}$$

$$= \frac{1}{\cot x}$$

$$= \boxed{\tan x}$$

#42: $\dfrac{\tan 2x + \cot 2\theta}{\sec 2\theta} = \csc 2\theta$

$$= \frac{\dfrac{\sin^2 2x}{\sin x \cos x} + \dfrac{\cos^2 2x}{\sin x \cos 2x}}{\sec^2 x}$$

$$= \frac{\dfrac{\sin^2 2x + \cos^2 2x}{\sin x \cos 2x}}{\sec^2 x}$$

$$= \frac{\sin^2 2x \cos^2 2x}{\sec 2x^2}$$

$$= \frac{\frac{1}{\sin^2 2x} \cdot \frac{1}{\cos^2 2x}}{\sec^2 2x}$$

$$= \frac{\csc^2 2x \cdot \sec^2 2x}{\sec^2 2x}$$

$$= \csc^2 2x$$

#56:

$(\cot^2 x + 1)(\sin^2 x + 1) = \cot^2 x + 2$

$= \csc^2 x (\sin^2 x + 1)$

$= \csc^2 x (\sin^2 x + \sin^2 x + \cos^2 x)$

$= 1 + 1 + \cot^2 x$

$= \cot^2 x + 2$

10. $\cot^2 x - \sin^2 x = 2\cos^2 x - 1$

$= \cot^2 x - (1 - \cos^2 x)$

or

$= \cot^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

37. $\tan t + \dfrac{\cos t}{1 + \sin t} = \sec t$

$= \dfrac{\sin t}{\cos t} + \dfrac{\cos t}{1 + \sin t}$

$= \dfrac{1 + \sin t}{\cos t + \cos t \sin t} + \dfrac{\cos^2 t}{\cos t + \cos t \sin t}$

$= \dfrac{1 + \sin^2 t + \cos^2 t}{\cos t + \cos t \sin t}$

$= \dfrac{1}{\cos t}$

$= \sec t$

56. $\dfrac{\csc x - \sec x}{\csc x + \sec x} = \dfrac{\cos x - 1}{\cot x + 1}$

$= \dfrac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}}$

$= \dfrac{\frac{\cos}{\sin \cos} - \frac{\sin}{\sin \cos}}{\frac{\cos}{\sin \cos} + \frac{\sin}{\sin \cos}}$

$= \dfrac{\cos - \sin}{\sin \cos} \cdot \dfrac{\sin \cos}{\cos + \sin}$

$= \dfrac{\cos - \sin}{\cos + \sin} \cdot \dfrac{\frac{1}{\sin}}{\frac{1}{\sin}}$

$= \dfrac{\frac{\cos}{\sin} - \frac{\sin}{\sin}}{\frac{\cos}{\sin} + \frac{\sin}{\sin}}$

$= \dfrac{\cot x - 1}{\cot x + 1}$

50. $\dfrac{\cos^2 x + 2\cos x + 1}{\cos x + 1} = 1 + \cos x$

$= \dfrac{(\cos x + 1)(\cos x + 1)}{\cos x + 1}$

$= \cos x + 1$

$= 1 + \cos x$

$= 1 + \dfrac{2\cos x}{2} + \dfrac{\cos^2 x}{2} - \dfrac{\sin^2 x}{2}$

$= 1 + \dfrac{2\cos x}{2} + \dfrac{\cos^2 x}{2} - \dfrac{1 - \cos^2 x}{2}$

$= 1 + \dfrac{2\cos x}{2} + \dfrac{\cos^2 x}{2} - \dfrac{1}{2} + \dfrac{\cos^2 x}{2}$

$= 1 + \cos x + \cos^2 x - \dfrac{1}{2}$

$= \dfrac{1}{2} + \cos x + \cos^2 x$

$= \dfrac{1}{2} + \cos x + \dfrac{2\cos^2 x}{2}$

$= \dfrac{1}{2} + \cos x + \dfrac{1 + \cos 2x}{2}$

$= \dfrac{1}{2} + \cos x + \dfrac{1}{2} + \dfrac{\cos 2x}{2}$

$= 1 + \cos x + \dfrac{\cos 2x}{2}$

showing

Sophie Caillier

25. $\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = 1$

$$\frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}} =$$

$$\frac{\sin t}{1} \cdot \frac{\sin t}{1} + \frac{\cos t}{1} \cdot \frac{\cos t}{1} =$$

$$\boxed{\sin^2 t + \cos^2 t = 1}$$

20. $\frac{\sec^2 t}{\tan t} = \sec t \cdot \csc t$

$$\frac{\cancel{\cos t}}{\sin t} \cdot \frac{1}{\frac{\cos t}{\sin t}} = \frac{1}{\sin t \cdot \cos t} = \boxed{\sec t \cdot \csc t}$$

45. $(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 =$$

$$\left(\frac{1-\sin x}{\cos x}\right)^2 =$$

$$\frac{1-\sin^2 x}{\cos^2 x} =$$

$$\boxed{\frac{1-\sin^2 x}{1+\sin^2 x}}$$

59. $\frac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

$$\frac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \boxed{\cos^2 x}$$

Alex

23. $\frac{1-\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$

$\csc\theta - \cot\theta = \csc\theta - \cot\theta$ ✓

18. $(\cos t)'(\cot t) = \frac{1-\sin^2 t}{\sin t}$

$(\cos t)\left(\frac{\cos t}{\sin t}\right) =$

$\frac{\cos^2 t}{\sin t} =$

$\frac{1-\sin^2 t}{\sin t} = \frac{1-\sin^2 t}{\sin t}$ ✓

44. $\frac{\cot x + \cot y}{1 - \cot x \cot y} = \frac{\cos y \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}$

$\frac{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}}{\left(\frac{\sin x \sin y}{\sin x \sin y}\right) \cdot 1 - \frac{\cos x}{\sin x} \cdot \frac{\cos y}{\sin y}} =$

$\frac{\frac{\sin y}{\sin y}\cdot\frac{\cos x}{\sin y} + \frac{\cos y}{\sin y}\cdot\frac{\sin x}{\sin x}}{\frac{\sin x \sin y - \cos x \cos y}{\sin x \sin y}} =$

$\frac{\sin y \cos x + \cos y \sin x}{\sin y \sin x} \cdot \frac{\sin x \sin y}{\sin x \sin y - \cos x \cos y} =$

$\frac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y} = \frac{\cos x \sin y + \sin x \cos y}{\sin x \sin y - \cos x \cos y}$ ✓

58. $(3\cos\theta - 4\sin\theta)^2 + (4\cos\theta + 3\sin\theta)^2 = 25$

$9\cos^2\theta - \cancel{24\cos\theta\sin\theta} + 16\sin^2\theta + 16\cos^2 + \cancel{24\cos\theta\sin\theta} + 9\sin^2\theta =$

$25\cos^2\theta + 25\sin^2\theta =$

$25(\cos^2\theta + \sin^2\theta) =$

$25 \cdot 1 =$

$25 = 25$ ✓

17. $(\sin T)(\tan T) = \frac{1-\cos^2 T}{\cos^2 T}$

$= \frac{\sin T}{1} \cdot \frac{\sin T}{\cos T}$

$= \frac{\sin^2 T}{\cos T}$

$= \boxed{\frac{1-\cos^2 T}{\cos T}}$

34. $(\csc^2 X)(\sec X) = \sec X + (\csc X)(\cot X)$

$= (\cot^2 X + 1)(\sec X)$

$= (\cot^2 X)(\sec X) + \sec X$

$= \frac{\cos^2 X}{\sin^2 X} \cdot \frac{1}{\cos X} + \sec X$

$= \frac{\cos X}{\sin^2 X} + \sec X$

$= \frac{\cos X}{\sin X} \cdot \frac{1}{\sin X} + \sec X$

$= \boxed{(\cot X)(\csc X) + \sec X}$

55. $(\tan^2\theta + 1)(\cos^2\theta + 1) = \tan^2\theta + 2$

$= \sec^2\theta\,(\cos^2\theta + 1)$

$= \frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} + 1$

$= 1 + \frac{1}{\cos^2\theta}$

$= 1 + \sec^2\theta$

$= 1 + \tan^2\theta + 1$

$= \boxed{\tan^2\theta + 2}$

41. $\frac{\tan 2\theta + \cot 2\theta}{\csc 2\theta} = \sec 2\theta$

$= \frac{\left(\frac{\sin 2\theta}{\sin 2\theta \cos 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}\right)\left(\frac{\cos 2\theta}{\cos 2\theta}\right)}{\csc 2\theta}$

$= \frac{\frac{\sin^2 2\theta}{\sin 2\theta \cos 2\theta} + \frac{\cos^2 2\theta}{\sin 2\theta \cos 2\theta}}{\csc 2\theta}$

$= \frac{\frac{\sin^2 2\theta + \cos^2 2\theta}{\sin 2\theta \cos 2\theta}}{\csc 2\theta}$

$= \frac{\csc 2\theta \cdot \frac{1}{\sin 2\theta \cos 2\theta}}{\csc 2\theta}$

$= \frac{\frac{1}{\sin 2\theta} \cdot \frac{1}{\cos 2\theta}}{\csc 2\theta}$

$= \frac{\sin 2\theta}{1} \cdot \frac{1}{\sin 2\theta} \cdot \frac{1}{\cos 2\theta}$

$= \frac{1}{\cos 2\theta}$

$= \boxed{\sec 2\theta}$

Clare Hall

Ben

13) $\frac{\frac{\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}}{\csc\theta} =$

$= \frac{1}{\csc\theta}$

$= \sin\theta$

32) $= \frac{-\sin^2 x + \cos^2 x - 1}{\sin x(\cos x + 1)}$

$= \frac{-1 - 1}{\sin x(\cos x - 1)}$

$= 0$

39) $= \tan^2 2x + 1$

$= \sec^2 2x$

53) $\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= 2 - \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= 2 + \frac{-\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}$

$= 2 + \frac{-1}{\cos\theta\sin\theta}$

$= 2 - \sec\theta\csc\theta$

26. $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$

$= \frac{\sin t}{\frac{1}{1}} \div \frac{\cos t}{\frac{1}{\sin t}}$

$= \frac{\sin t}{\frac{\sin t}{\cos t}} \cdot \frac{\cos t}{\frac{1}{\sin t}} + \cot t \cdot \frac{\sin t}{\cot t}$

$= \cos t + \sin t$

22. $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

$= \frac{\csc^2 t - 1}{\csc t}$

$= \frac{\frac{1}{\sin^2 t} - \frac{\sin t}{\sin t}}{\frac{1}{\sin t}}$

$= \left(\frac{1}{\sin^2 t} - \frac{\sin t}{\sin t}\right) \cdot \frac{\sin t}{1}$

$= \frac{1}{\sin t} - \sin t$

$= \csc t - \sin t$

46. $(\csc x - \cot x)^2 = \frac{1-\cos x}{1+\cos x}$

$= \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)^2$

$= \frac{1-\cos x}{\sin x}\left(\frac{1-\cos x}{\sin x}\right)$

$\frac{(1-\cos x)^2}{\sin^2 x}$

$= \frac{(1-\cos x)(1-\cos x)}{(1-\cos^2 x)}$

$= \frac{(1-\cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$= \frac{1-\cos x}{1+\cos x}$

60. $\frac{\sin x + \cos x}{\sin x} - \frac{\cos x - \sin x}{\cos x} = \sec x \csc x$

$= \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}$

$= 1 + \frac{\cos x}{\sin x} - 1 + \frac{\sin x}{\cos x}$

$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x}{\cos x \sin x} + \frac{\sin^2 x}{\cos x \sin x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos x \sin x}$

$= \frac{1}{\cos x (\sin x)}$

$= \sec x \csc x$

Maggie

Sylvie Ashford

21 $\dfrac{\tan^2 t}{\sec t} = \sec t - \cos t$

$\dfrac{(\sec^2 t - 1)}{\sec t} =$

$\dfrac{\sec^2 t - 1}{1} \cdot \dfrac{\cos t}{1} =$

$(\sec^2 t - 1)(\cos t) =$

$(\sec t + 1)(\sec t - \cos t)(\cos t) =$

$\sec t - \cos t =$

16 $\cos^2\theta(1 + \tan^2\theta) = 1$

$\cos^2\theta(\sec^2\theta) = 1$

$\dfrac{\cos^2\theta}{1} \cdot \dfrac{1}{\cos^2\theta} =$

$\dfrac{\cos^2\theta}{\cos^2\theta} =$

$1 =$

43 $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)} =$

$\dfrac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{1 - \frac{\sin x \sin y}{\cos x \cos y}} =$

$\dfrac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}} =$

$\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \cdot \dfrac{\cos x \cos y}{\cos x \cos y - \sin x \sin y} =$

$\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} =$

57 $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = 2$

$(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\cos\theta + \sin\theta)(\cos\theta + \sin\theta) =$

$\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta =$

$2\cos^2\theta + 2\sin^2\theta =$

$2(\cos^2\theta + \sin^2\theta) =$

$2(1) =$

2